

Active Control of the Parametric Resonance in the Modified Rayleigh-Duffing Oscillator

C. H. Miwadinou*, A. V. Monwanou† and J. B. Chabi Orou‡

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Institut de Mathématiques et de Sciences Physiques, BP: 613 Porto Novo,
Bénin

Abstract

The present paper examines the active control of parametric resonance in modified Rayleigh-Duffing oscillator. We used the method of averaging to obtain steady-state solutions. We have found the critical value of the parametrical amplitude which indicates the boundary layer where the control is efficient in reducing the amplitude vibration. We find also the effects of excitation parameters and time-delay on dynamical of this system with the principal parametric resonance. We obtain also for this oscillators the Hopf bifurcation or saddle-node bifurcation for certain values of parametric parameters and time-delay and we have studied the influence of parameter k_2 which is one parameter which modify the ordinary Rayleigh-Duffing oscillator. We have discussed the appropriate choice of the time-delay and control gain. We finally studied the stability of fixed point and it is found that the appropriate choice of the time-delay can broaden the stable region of the non-trivial steady-state solutions enhance the control performance. Numerical simulations are performed in order to confirm the analytically results.

keywords: Active control, parametric resonance, primary, modified Rayleigh-Duffing oscillator, stability, Hopf burfication, amplitude vibration.

1 Introduction

In recent years, a twofold interest has attracted theoretical, numerical, and experimental investigations to understand the behavior of nonlinear oscillators. Theoretical (fundamental) investigation reveals their rich and complex behavior, and the experimental (self-excited oscillators) describes the evolution of many biological, chemical, physical, mechanical, and industrial systems [1, 2, 3]. Recently, the

*clement.miwadinou@imsp-uac.org

†movins2008@yahoo.fr

‡Author to whom correspondence should be addressed: jchabi@yahoo.fr

chaotic behavior of these oscillators is exploited in the field of communication for coding information [4]. The considerable efforts have been devoted to the control of vibrating structures in various fields of fundamental and applied sciences. Two major aims in the scope of the researchers are: reduce the amplitude of vibrations, inhibit chaos and escape from potential well. Applications are well encountered in structural mechanics (see Refs. [1–5]). Parametric excitation occurs in a wide variety of engineering application (Refs. [10–14]). Among various types of control strategies, the active feedback control usually developed by means of electromagnetic force or by a mechanical device (Ref.[5–9]) and the parametric. In [15] Xin-Ye and al demonstrated that in the dynamical behaviour of a parametrically excited Duffing–Van der Pol oscillator under linear-plus-nonlinear state feedback control with a time-delay, nontrivial steady state responses may lose their stability by a saddle-node or Hopf bifurcation as parameters vary. M.Siewe Siewe and al [16] studied the dynamics of a parametrically excited Rayleigh–Duffing oscillator state feedback control with a time-delay, nontrivial steady state solutions may lose their stability by a saddle-node or Hopf bifurcation as parameters vary.

Our objective of this paper is to control the amplitude vibration of a parametrically excited modified Rayleigh-Duffing oscillator with time-delayed feedback position and linear velocity terms. The modified Rayleigh-Duffing oscillator that we consider in this work is governed by following equation:

$$\ddot{x} + \epsilon\alpha\dot{x} + \epsilon\beta\dot{x}^2 + \epsilon\gamma\dot{x}^3 - 2\epsilon k_1\dot{x}x + \epsilon k_2\dot{x}^2x + (\delta + \epsilon a \cos 2\omega_0 t)x + \epsilon\lambda x^3 = 0. \quad (1)$$

We used the method of averaging for study this oscillator. The amplitude peak of the parametric resonance can be reduced by means of a correct choice of the time-delay and the feedback gain. We discussed how the existence region of steady-state solutions is modified by the feedback control and we showed the existence regions of the nontrivial solution in the plane of the parametric excitation amplitude and the detuning parameter for an uncontrolled system, a controlled system without time-delay, and those with time-delays corresponding to the minimum and maximum values of an appropriate equivalent damping. A bifurcation analysis and parametric excitation-response and frequency-response curves are presented. We found the effect of k_2 on the control. Then we performed a stability analysis of the bifurcation of the model and we derived sufficient conditions for stable nontrivial solutions in order to exclude the presence of modulated motion.

2 Model and linear control

2.1 Model

Consider the modified Rayleigh-Duffing oscillator equation which describes the glycolytic reaction catalized by phosphofructokinase, namely the Selkov equations

and abstract trimolecular chemical reaction[17].

$$\frac{d^2\xi}{dt^2} + \tilde{\lambda}\frac{d\xi}{dt} + \tilde{\lambda}''\left(\frac{d\xi}{dt}\right)^2 + \tilde{\lambda}'\left(\frac{d\xi}{dt}\right)^3 + \Omega^2\left(1 - k\frac{d\xi}{dt}\right)^2\xi = 0 \quad (2)$$

Perturbing this system by Duffing force and parametric excitation force, we obtained the parametric dissipative modified Rayleigh-Duffing oscillator which is expressed by eq. (1). We consider a single-degree-of-freedom model with a nonlinear soft spring, a linear damper and a linear displacement feedback control system. The model is described by the following differential equation system:

$$\begin{aligned} \ddot{x} + \epsilon\alpha\dot{x} + \epsilon\beta\dot{x}^2 + \epsilon\gamma\dot{x}^3 - 2\epsilon k_1\dot{x}x + \epsilon k_2\dot{x}^2x + (\delta + \epsilon a \cos 2\omega_0 t)x \\ + \epsilon\lambda x^3 - \epsilon bx(t - \tau) - \epsilon c\dot{x}(t - \tau) = 0, \end{aligned} \quad (3)$$

where x, \dot{x}, \ddot{x} are displacement, velocity and acceleration respectively; ω_0 is the natural frequency, ϵ is a positive parameter, $\alpha, \beta, \gamma, k_1, k_2, \delta, \lambda, a$ are constants, b and c are the control gain parameters and τ is time-delay parameter. In this equation, $-\epsilon bx(t - \tau) - \epsilon c\dot{x}(t - \tau)$ is the feedback force in the system. The resonances appear when $\delta = \omega_0^2 + \epsilon\delta_1$.

2.2 Linear control

In this part, we used the method of averaging to study the eq. (3) and to research the controlling domain. In the absence of the parameter ϵ , eq.(3) reduces to

$$\ddot{x} + \omega_0^2 x = 0, \quad (4)$$

with the solution:

$$x = A \cos(\omega_0 t + \phi). \quad (5)$$

If $\epsilon \neq 0$, A and ϕ depend on time t . In this condition, differentiating (5) and using the method of averaging, we found that the eq. (3) is divided into:

$$\frac{dA}{dt} = -\epsilon F[A \cos(\omega_0 t + \phi), -A \sin(\omega_0 t + \phi)] \sin(\omega_0 t + \phi) \quad (6)$$

$$A \frac{d\phi}{dt} = -\epsilon F[A \cos(\omega_0 t + \phi), -A \sin(\omega_0 t + \phi)] \cos(\omega_0 t + \phi), \quad (7)$$

where

$$\begin{aligned} F = F(x, \dot{x}) = -\alpha\dot{x} - \beta\dot{x}^2 - \gamma\dot{x}^3 + 2k_1\dot{x}x - k_2\dot{x}^2x \\ -(\delta_1 + a \cos 2\omega_0 t)x - \lambda x^3 + bx(t - \tau) + c\dot{x}(t - \tau). \end{aligned} \quad (8)$$

When applying the hypothesis of the method of averaging (\dot{A} and $\dot{\phi}$ are small for one period for small ϵ), we obtain

$$\frac{dA}{dt} = -\frac{\epsilon}{T_0} \int_0^{T_0} F[A \cos(\omega_0 t + \phi), -A \sin(\omega_0 t + \phi)] \sin(\omega_0 t + \phi) dt \quad (9)$$

$$A \frac{d\phi}{dt} = -\frac{\epsilon}{T_0} \int_0^{T_0} F[A \cos(\omega_0 t + \phi), -A \sin(\omega_0 t + \phi)] \cos(\omega_0 t + \phi) dt \times$$

$$\cos(\omega_0 t + \phi) dt, \quad (10)$$

where T_0 is the natural period of this oscillator. Using x and his derivative, eq.(8) becomes

$$\begin{aligned} F[A \cos(\omega_0 t + \phi), -A \sin(\omega_0 t + \phi)] &= \alpha \omega_0 A \sin(\omega_0 t + \phi) \\ &- \beta \omega_0^2 A^2 \sin^2(\omega_0 t + \phi) + \gamma \omega_0^3 A^3 \sin^3(\omega_0 t + \phi) + 2k_1 \omega_0 A^2 \sin(\omega_0 t + \phi) \times \\ &\cos(\omega_0 t + \phi) - k_2 A^3 \omega_0^2 \sin^2(\omega_0 t + \phi) \cos(\omega_0 t + \phi) \\ &- (\delta_1 + a \cos 2\omega_0 t) A \cos(\omega_0 t + \phi) - \lambda A^3 \cos^3(\omega_0 t + \phi) \\ &+ b \tilde{A} \cos[\omega_0(t - \tau) + \tilde{\phi}] - c \omega_0 \tilde{A} \sin[\omega_0(t - \tau) + \tilde{\phi}], \end{aligned} \quad (11)$$

where $\tilde{A} = A(t - \tau)$ and $\tilde{\phi} = \phi(t - \tau)$.

Substituting (11) into the eq.(9) and eq.(10) and evaluating the integrals, we find

$$\begin{aligned} \frac{dA}{dt} &= -\frac{\epsilon}{4} [\omega_0 A (2\alpha + \frac{3}{2} \gamma \omega_0^2 A^2) - a A \sin 2\phi \\ &+ 2b \tilde{A} \sin(\phi - \tilde{\phi} + \omega_0 \tau) - 2c \omega_0 \tilde{A} \cos(\phi - \tilde{\phi} + \omega_0 \tau)], \end{aligned} \quad (12)$$

$$\begin{aligned} A \frac{d\phi}{dt} &= \frac{\epsilon}{4} [2\delta_1 A + \frac{1}{2} (k_2 + 3\lambda) A^3 \omega_0^2 + a A \cos 2\phi \\ &- 2b \tilde{A} \cos(\phi - \tilde{\phi} + \omega_0 \tau) + 2c \omega_0 \tilde{A} \sin(\phi - \tilde{\phi} + \omega_0 \tau)]. \end{aligned} \quad (13)$$

Eqs.(12) and (13) show that A and ϕ are $O(\epsilon)$. Expanding in Taylor series $\tilde{A}, \tilde{\phi}$:

$$\tilde{A} = A(t - \tau) = A(t) - \tau \dot{A}(t) + \tau^2 \ddot{A}(t) + \dots, \quad (14)$$

$$\tilde{\phi} = \phi(t - \tau) = \phi(t) - \tau \dot{\phi}(t) + \tau^2 \ddot{\phi}(t) + \dots \quad (15)$$

Eqs.(14) and (15) indicate that we can replace \tilde{A} and $\tilde{\phi}$ by A and ϕ in eq.(12) and (13) since $\dot{A}, \dot{\phi}$ and $\ddot{A}, \ddot{\phi}$ are $O(\epsilon)$ and $O(\epsilon^2)$ respectively. This reduces an infinite-dimensional problem in functional analysis to a finite-dimensional problem by assuming that the product $\epsilon\tau$ is small [16]. By setting $t = \tilde{t}/\epsilon$ as the new time scale, we have the following averaged equations:

$$\begin{aligned} \dot{A} &= \frac{A}{4} [\omega_0 (-2\alpha - \frac{3}{2} \gamma \omega_0^2 A^2) + a \sin 2\phi - 2b \sin \omega_0 \tau \\ &+ 2c \omega_0 \cos \omega_0 \tau], \end{aligned} \quad (16)$$

$$\begin{aligned} A \dot{\phi} &= \frac{A}{4} [2\delta_1 + \frac{1}{2} (k_2 + 3\lambda) A^2 \omega_0^2 + a \cos 2\phi - 2b \cos \omega_0 \tau \\ &+ 2c \omega_0 \sin \omega_0 \tau]. \end{aligned} \quad (17)$$

Seeking steady state, we have $\dot{A} = 0$ and $A \dot{\phi} = 0$ in (16) and (17). A trivial solution is $A = 0$ and we find that the non trivial solutions verify the equation:

$$A_0^4 - 2P A_0^2 + Q = 0, \quad (18)$$

where the quantities P and Q are given by

$$P = \frac{1}{\frac{9}{4}\gamma^2\omega_0^6 + \frac{1}{4}(k_2 + 3\lambda)^2\omega_0^4} \left[\frac{3}{2}\gamma\omega_0^3(-2\omega_0\alpha - 2b\sin\omega_0\tau + 2c\omega_0\cos\omega_0\tau) - \frac{1}{2}\omega_0^2(k_2 + 3\lambda)(2\delta_1 - 2b\cos\omega_0\tau + 2c\omega_0\sin\omega_0\tau) \right], \quad (19)$$

$$Q = \frac{1}{\frac{9}{4}\gamma^2\omega_0^6 + \frac{1}{4}(k_2 + 3\lambda)^2\omega_0^4} \left[(-2\omega_0\alpha - 2b\sin\omega_0\tau + 2c\omega_0\cos\omega_0\tau)^2 + (2\delta_1 - 2b\cos\omega_0\tau + 2c\omega_0\sin\omega_0\tau)^2 - a^2 \right]. \quad (20)$$

Indeed P and Q can be rewritten as:

$$P = \frac{4}{\omega_0^2[9\gamma^2\omega_0^2 + (k_2 + 3\lambda)^2]} \left[(bk_2 + 3\lambda b + 3c\gamma\omega_0^2)\cos\omega_0\tau - (c\omega_0k_2 + 3c\lambda\omega_0 + 3\gamma b\omega_0)\sin\omega_0\tau - 3\gamma\alpha\omega_0^2 - (k_2 + 3\lambda)\delta_1 \right], \quad (21)$$

$$Q = \frac{4}{\omega_0^4[9\gamma^2\omega_0^2 + (k_2 + 3\lambda)^2]} \left[K - 8\omega_0bc\sin 2\omega_0\tau + 8\omega_0(c\delta_1 + \alpha b)\sin\omega_0\tau - 8(b\delta_1 + c\alpha\omega_0^2)\cos\omega_0\tau \right], \quad (22)$$

with

$$K = 4\omega_0^2\alpha^2 + 4\omega_0^2c^2 + 4\delta_1^2 + 4b^2 - a^2. \quad (23)$$

Now, solving the equation (18), we obtain

$$A_0 = \sqrt{P \pm \sqrt{P^2 - Q}} \quad (24)$$

with the conditions $P > 0, Q > 0, P^2 > Q$ or $Q < 0$.

We denoted by A_{0c} and A_{0u} the amplitude of the controlled system (i.e. when $b \neq 0$ and $c \neq 0$) and the amplitude of the uncontrolled system (i.e. when $b = 0$ and $c = 0$) respectively. In the boundary of parameter space where the control of the amplitude is efficient, $A_{0u} = A_{0c}$ [16]. Using these conditions, from eq.(24), we find the boundary separating the domain where the control is efficient from the domain where the control is inefficient which is given by

$$a_c^2 = \frac{1}{(P_c - P_u)^2} \left[(4\omega_0^2\alpha^2 + 4\delta_1^2)(P_c - P_u)^2 - Q_1(P_u^2 + P_cP_u) + \frac{Q_1^2}{\omega_0^4[9\gamma^2\omega_0^2 + (k_2 + 3\lambda)^2]} \right], \quad (25)$$

where Q_1 is defined by

$$Q_1 = 4b^2 + 4\omega_0^2c - 8\omega_0bc\sin 2\omega_0\tau + 8\omega_0(c\delta_1 + \alpha b)\sin\omega_0\tau - 8(b\delta_1 + c\alpha\omega_0^2)\cos\omega_0\tau, \quad (26)$$

where P_c and P_u represent the function corresponding to the controlled and uncontrolled system.

To validate our technical control in reducing the amplitude vibration, we have simulated numerically the set of eq.(25). We have plotted in Fig.1 for two conditions $c > b$ and $b > c$, the domain in the space parameters (τ, a_c) where the control is efficient in reducing the amplitude of the oscillations. (a) represents the case where $k_2 = 0$ and (b) corresponds to $k_2 \neq 0$. We remark that it is clear that the critical value of amplitude excitation a is lower in the case where the control gain parameters verify the inequality $c > b$. We therefore conclude that this domain is reduces with k_2 . The Fig.2 represents the frequency- and force-response curves resulting from eq.(24) and the results obtained in Fig.1 are clearly verified. We see that the steady-state response of the system is lower when the parameter a is below the boundary domain obtained in Fig.1(b). Now we have plotted in Fig.3 the surface amplitude for two conditions for b , where c is fixed. (a) corresponds to $b = 0.3, c = 0.5$ while for (b), $b = 0.7, c = 0.5$. We noticed from Fig.3(a) that the maximum value of the amplitude of steady-state solutions are lower compared to the one in Fig.3(b). This differents simulations prove that our technical are confirmed. From Fig.3, one can see that there are range of time delay τ and detuning parameters δ_1 where the steady-state solution is zero. This can be a consequence of noise effect.

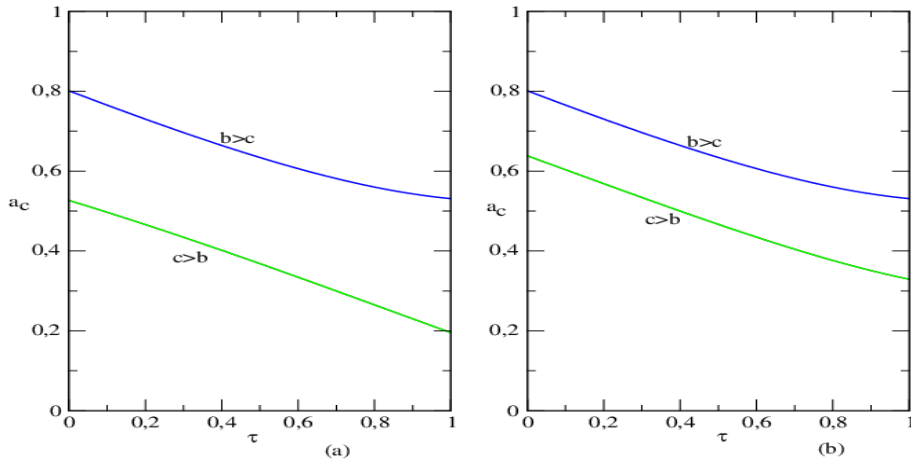


Figure 1: Boundary criterion for the effectiveness of control of the oscillation amplitude in the space (a_c, τ) for (a) $k_2 = 0$ and (b) $k_2 = 0.1$; blue curve corresponding to $b = 0.7$ and green curve corresponding to $b = 0.3$, the other parameters are $\gamma = 0.002$; $\alpha = -0.002$; $\lambda = -1$; $\delta_1 = 0.01$; $c = 0.5$.

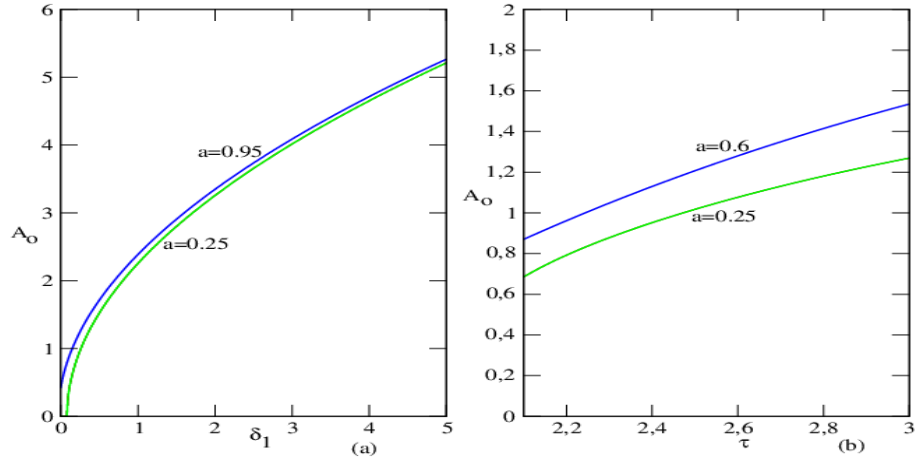


Figure 2: The effect of reducing the steady-state response with control parameter a obtained in Fig.1 (b). (a) Frequency-response and (b) force-response.

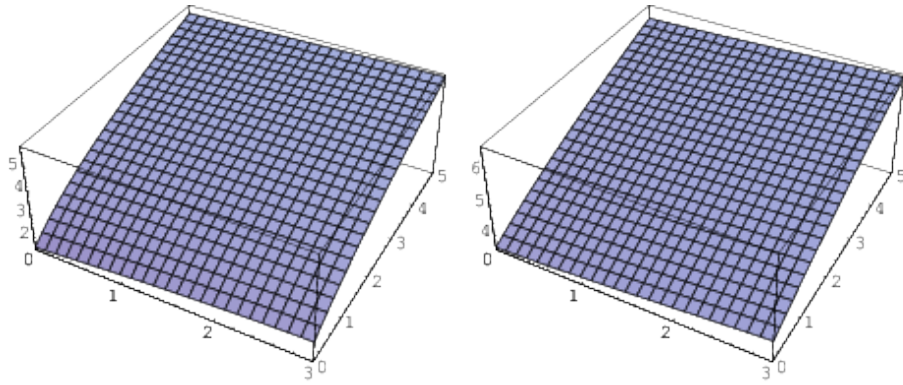


Figure 3: Time-delayed position gain b effect on response-surface curves given by Eq. (22): left curve corresponding to $b = 0.3$ and right curve corresponding to $b = 0.7$.

3 Effects of the parameters in presence

In this section, we study separately the effects of gain parameters, parametric excitation, and combined gain parameters and parametric excitation in the response from eq.(24) of our model amplitude.

3.1 Effects of parametric excitation alone with null value of τ

The effects of parametric excitation are found from eq.(24) with times delay equal zero. We have plotted in Fig.4, the frequency- and forces-responses of the non trivial solution for $b \neq 0$ and $c \neq 0$. For each values chosen for amplitude or frequency of parametric excitation, we plotted two curves. In Fig.4(a), we see the limits of curves correspond to stable and unstable for the nontrivial solution. For each value of a , the two curves are the same from certain values of δ_1 . We noticed also that for fixed parametric amplitude, two symetric branches appear around the point near the origin. The reduction effect due to the parameter a is quite important and is found to be visible only in a small region of detuning parameter surrounding this point. The Fig.4(b) represents the force-response for two differents values of δ_1 . It is clear that the Hopf bifurcation with multi-solution occurs as the detuning parameter δ_1 increases. We have also found that the saddle-node bifurcation disappears for lower values of δ_1 .

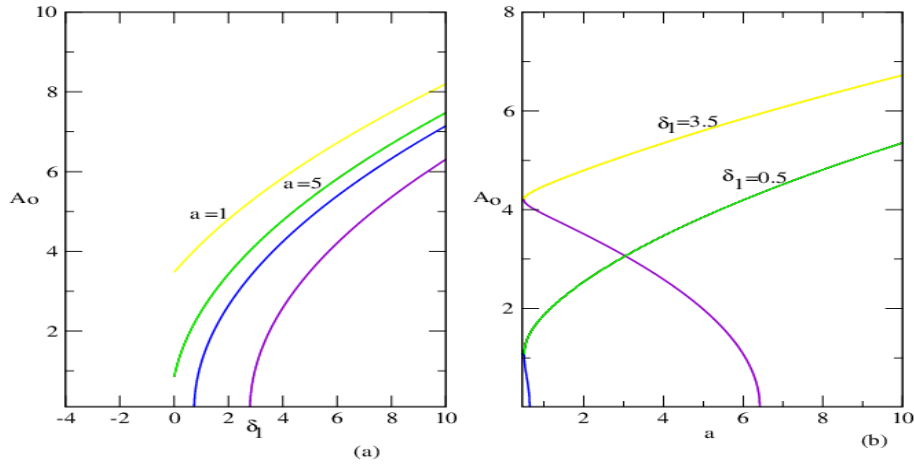


Figure 4: (a) The effect of the parametric amplitude in the frequency-response of the system in the absence of the time-delay. (b) Force-response for two detuning parameter values. Here $b = 0.3, \gamma = 0.002, \alpha = -0.002, \lambda = -1, k_2 = 0.1$, and $c = 0.5$.

3.2 Effects of control gain parameters for a equal to zero

In this part we have found the control gain parameters effects when the parametric excitation a equal to zero. This Fig.5 and Fig.6 illustrated the frequency-response of the system for different situations for fixed parameters. (a) and (b) represent for each figure the frequency-response of system respectively for $a = 0, c = 0$ and $a = 0, c = 0$. In Fig.5 $\tau = 0.008, \gamma = 0.8, \alpha = -0.8, k_2 = 0.1$ and Fig.6 $\tau = 0.008, \gamma = 0.002, \alpha = -0.002, k_2 = 0.8$. We obtain in each case of this figure that unstable and stable solutions curves are the same. We have also found that the two control gain parameters increases the region of the detuning parameter which is more visible when the time-delayed position is considered alone than the case where the time-delayed velocity is considered alone. Fig.5 compared to Fig.6, prove that the self-excitation parameters α, γ and k_2 affected also the peak of amplitude of vibration which is greater in the case of time-delayed position.

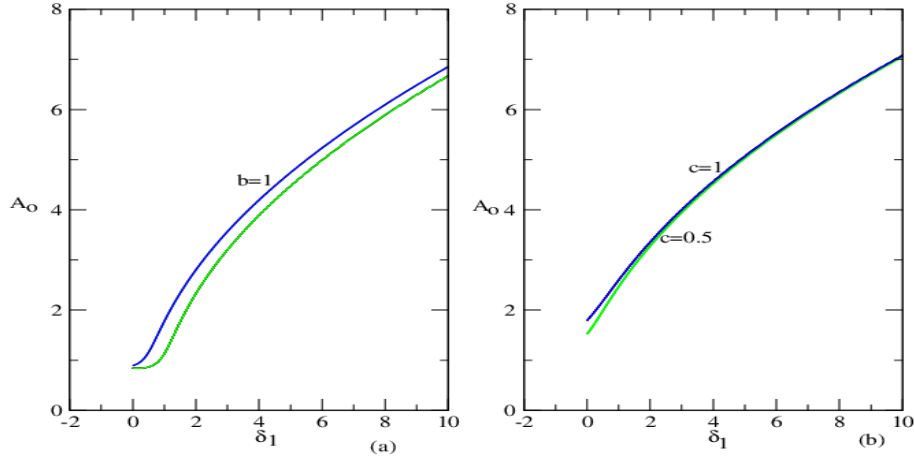


Figure 5: (a) The effect of the position time-delay alone in the frequency-response of the system without parametric excitation and (b) the effect of the velocity time-delay alone in the frequency-response of the system without parametric excitation for $\tau = 0.008, \gamma = 0.8, \alpha = -0.8$ and $k_2 = 0.1$.

3.3 Effects of combined parametric excitation and control gain parameters

In this part, we have plotted in space the (A_0, δ_1) and (A_0, a) corresponding respectively to frequency-response and force-response for non-zero time delay, $\gamma = 0.8, \alpha = -0.8, k_2 = 0.8$. This Fig.7 are used to analysis the effects of combined parametric excitation and control gain parameters. We have developed the similar comments obtained in Fig.4 but the domain of different bifurcations is not the same.

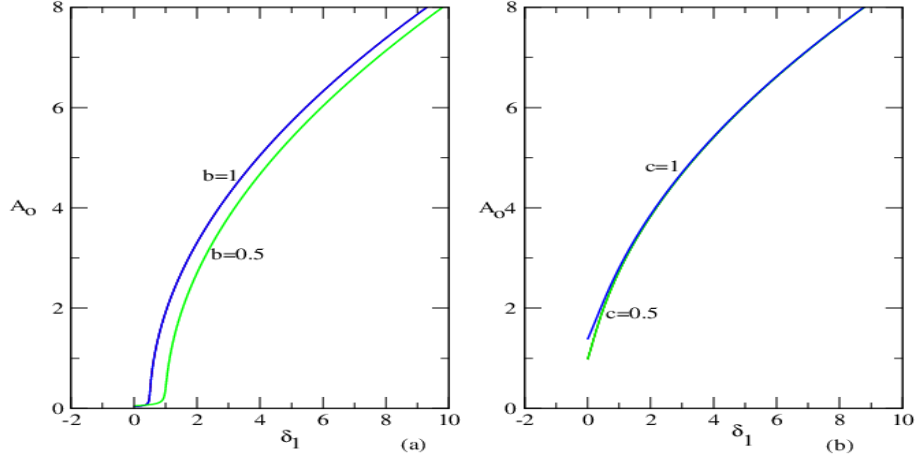


Figure 6: (a) The effect of the position time-delay alone in the frequency-response of the system without parametric excitation and (b) the effect of the velocity time-delay alone in the frequency-response of the system without parametric excitation for $\tau = 0.008$, $\gamma = 0.002$, $\alpha = -0.002$ and $k_2 = 0.8$.

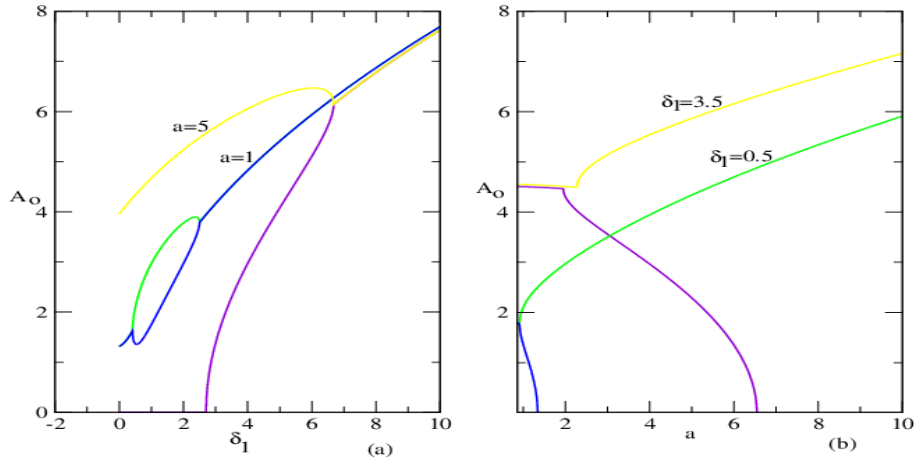


Figure 7: Effect of parametric perturbation added to the time-delayed term in the response of the system. (a) Frequency-response amplitude and (b) Force-response. The values of the parameters are $\tau = 0.008$, $\gamma = 0.8$, $\alpha = -0.8$, $k_2 = 0.8$, $b = 0.3$, $\lambda = -1$, and $c = 0.5$.

4 Stability analysis of solutions

In this case, consider the equilibrium points defined by eq.(24). To study the behavior of the steady state, we resort to the linearized stability principle by using the Routh-Hurwitz criterion [18]. We apply a little perturbation (with $u \ll A_0$ and $v \ll \phi_0$):

$$\begin{cases} A = A_0 + u \\ \phi = \phi_0 + v \end{cases} \quad (27)$$

We obtain eigenvalues equation of the Jacobian matrix defined at (A_0, ϕ_0) of the corresponding system equations (16) and (17)

$$\eta^2 + T\eta + D = 0, \quad (28)$$

where T is the opposite of the trace of the Jacobian matrix and D the determinant of the Jacobian matrix. T and D are defined by:

$$T = \frac{1}{4}[-4\omega_0\alpha + 3\gamma\omega_0^3 A_0(1 + A_0) - 4b \sin \omega_0\tau + 4c\omega_0 \cos \omega_0\tau], \quad (29)$$

$$D = A_0 D_0 = A_0(\kappa_2 A_0^4 + \kappa_1 A_0^2 + \kappa_0), \quad (30)$$

$$\kappa_2 = \frac{3}{8}\omega_0^2[9\gamma^2\omega_0^2 + \frac{1}{4}(k_2 + 3\lambda)^2], \quad (31)$$

$$\begin{aligned} \kappa_1 = & \chi + 4\omega_0(3b\gamma\omega_0 + c\omega_0^2(k_2 + 3\lambda)) \sin \omega_0\tau \\ & - \omega_0^2(9c\gamma\omega_0 + 4b(k_2 + 3\lambda)) \cos \omega_0\tau, \end{aligned} \quad (32)$$

$$\begin{aligned} \kappa_0 = & K - 8bc\omega_0 \sin 2\omega_0\tau - 8(b\delta_1 + c\alpha\omega_0^2) \cos \omega_0\tau \\ & + 8(b\alpha\omega_0 + c\omega_0\delta_1) \sin \omega_0\tau \end{aligned} \quad (33)$$

and

$$\chi = 3\omega_0^4\alpha\gamma + 9\gamma\alpha\omega_0^3 + 4\delta_1(k_2 + 3\lambda)\omega_0^2 - 3c\gamma\omega_0^3. \quad (34)$$

Equation (28) has in general two roots:

$$\eta_{\pm} = \frac{1}{2}(-T \pm \sqrt{T^2 - 4D}) \quad (35)$$

A positive real root indicates an unstable solution, whereas if the real parts of the eigenvalues are all negative then the steady-state solution is stable. When the real part of an eigenvalue is zero, a bifurcation occurs. A change from complex roots with negative real parts to complex roots with positive real parts would indicate the presence of a supercritical or subcritical Hopf bifurcation. The question of which possibility actually occurs depends on the nonlinear terms. The Routh-Hurwitz criterion implies that the steady-state response is asymptotically stable if and only

if $T > 0$ and $D > 0$ (i.e. $D_0 > 0$ because amplitude A_0 are positive) which keep the real parts of eigenvalues negative. We obtained

$$4b \sin \omega_0 \tau + 4c \omega_0 \cos \omega_0 \tau > 4\omega_0 \alpha - 3\gamma \omega_0^3 A_0 (1 + A_0) \quad (36)$$

$$\begin{aligned} a^2 = & 4\omega_0^2 \alpha^2 + 4\delta_1^2 + 4b^2 + 4\omega_0^2 c^2 - 8cb\omega_0 \sin 2\omega_0 \tau - \\ & 8(b\delta_1 + c\alpha\omega_0^2) \cos \omega_0 \tau + 8(b\alpha\omega_0 + c\omega_0\delta_1) \sin \omega_0 \tau + \\ & \kappa_2 A_0^4 + \kappa_1 A_0^2 \end{aligned} \quad (37)$$

From these conditions, when parametric excitation and time-delayed position (resp. time-delayed velocity) are combined, the stability domain in space (δ_1, a) of the non-trivial solutions A_0 is obtained in Fig.8, where (a) correspond to the case $b < c$ and (b) correspond to the case $b > c$. We noticed clearly that for each case, the plane is divided in three regions where one of them is unstable and two are stable. Other remark is that the stable domain increases when the time-delay control gain position is greater than the time-delay control gain velocity. These two main conclusions confirm our results obtain in the linear control part and in the effect of differents part.

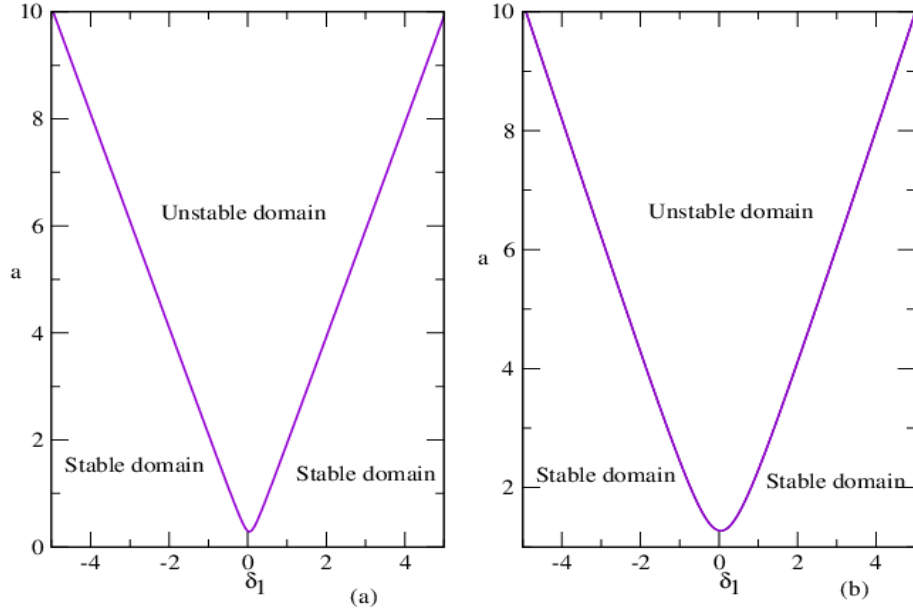


Figure 8: Stability of non-trivial solution in the space (δ_1, a) with $\alpha = -0.1, \gamma = 0.1, c = 0.5, k_2 = 0.1$, and $\tau = \pi$. (a) $b = 0.3$ and (b) $b = 0.7$

5 Conclusion

In this paper we have investigated the interaction of parametric excitation with time-delayed position and velocity on the parametric resonance. By the averaging method to first order, we found the equilibrium point of the system giving the amplitude and phase of vibration of the system starting. This study shows the area in which the amplitude of the vibration of the oscillator is reduced i.e. the area where vibration control is effective. We obtain also for this oscillator the Hopf bifurcation or saddle-node bifurcation for certain values of parametric parameters and time-delay and we have studied the influence of parameter k_2 which is one parameter which modify the ordinary Rayleigh-Duffing oscillator. We applied the Routh-Hurwitz criterion for the stability study of steady-state response and we obtain the stability domain of the parametric oscillator modified Rayleigh-Duffing. We have shown clearly that the amplitude of the vibration at the primary resonance can therefore be controlled by the active control. Finally, the effects of various parameters on the control of parametric resonance have been studied.

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